# 6 TIME & DISTANCE

**TIP**

**Conversion:**

1 km/h = $\frac{5}{18}$ m/s or

1 m/s = $\frac{18}{5}$ km/h

*Do you know?*

The average velocity is **NOT** = Sum of the two velocities/ 2.
If distance covered remains same then **Average speed = H.M. of velocities.**

1. The fundamental relationship between Distance(s), time(t) and speed(v) is given by: **s = v x t.**

2. Let $v_1$ and $v_2$ be the velocity of the two vehicles and let $v_1 > v_2$
If both the vehicles are moving in the same direction then their
Relative Velocity = R.V. = $v_1 - v_2$
If both the vehicles are moving in the opposite direction then their
Relative velocity = R.V. = $v_1 + v_2$

3. Length of the objects is always added whether they are moving in the same direction or opposite direction. If $L_1$ and $L_2$ are the length of the two objects then,
Total length = $L_1 + L_2$

4. Average velocity = Total distance covered/ Total time taken
If $x_1$ & $x_2$ are the distances covered at velocities $v_1$ & $v_2$ respectively then the average velocity over the entire distance $(x_1 + x_2)$ is given by $\dfrac{x_1 + x_2}{\frac{x_1}{v_1} + \frac{x_2}{v_2}}$.

5. A man travels first $\frac{1}{2}$ of the distance at a velocity $v_1$, second $\frac{1}{2}$ of the distance at a velocity $v_2$ then, Average velocity = $\dfrac{2v_1v_2}{v_1 + v_2}$ = H.M of the 2 velocities.

6. A man covers first $\frac{1}{3}$ of the distance at $v_1$, second $\frac{1}{3}$ of the distance at $v_2$ & the third $\frac{1}{3}$ of the distance at $v_3$, then Average velocity = $\dfrac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$

## Trains

> **Do you know?**
>
> If a speed of a body is changed in the ratio x : y, then ratio of the time taken changes in the ratio y : x.

(i) When a train approaches a stationary object (a tree, a stationary man, a lamp-post; we assume the length of the object to be infinitely small; provided its length isn't mentioned), the time taken by the train to cross such an object is the same as the time taken by the train to cross a distance equal to its own length at its own velocity.

$\therefore$Time taken by the train to cross that object = length of the train / velocity of the train.

(ii) However, when a train approaches a platform, the time taken by the train to cross the platform is same as the time taken by the train to cross a distance equal to its own length plus the length of the platform at its own velocity.

$\therefore$Time taken by the train to completely cross the platform = (Length of the train + length of the platform) / velocity of the train.

(iii) For two trains having lengths $l_1$ & $l_2$ and traveling in the same direction with velocities $v_1$ & $v_2$ respectively ($v_1 > v_2$).

Time taken to cross each other completely = total distance/relative velocity

$$= \frac{l_1 + l_2}{v_1 - v_2}$$

(iv) Similarly, for two trains traveling in the opposite direction :

Time taken to cross each other completely = total distance /relative velocity

$$= \frac{l_1 + l_2}{v_1 + v_2}.$$

(v) Train crosses the persons moving at speed $V_1$ & $V_2$ in time $t_1$ & $t_2$ respectively. Then, the length of the train is given by

$$L = (V_1 - V_2) \times \frac{t_1 \times t_2}{t_1 - t_2} \text{ or Difference of speed} \times \frac{\text{Multiplication of time}}{\text{Difference of time}}$$

(vi) If two trains start at the same time from two points X & Y and move towards each other and after crossing they take a & b hrs respectively to reach Y and X, then

$$\frac{\text{x's speed}}{\text{y's speed}} = \sqrt{\frac{b}{a}}.$$

Example

***A train 110 m long travels at 60 kmph. How long does it take***

(a) to pass a telegraph post by the side of the track?

(b) to pass a man running at 6 kmph in the same direction as the train?

(c) to pass a man running at 6 kmph in the opposite direction?

(d) to pass a station platform 240 m long?

(e) to pass another train 170 m long, running at 40 kmph in the same direction?

(f) to pass another train 170 m long, running at 60 kmph in the opposite direction?

**Sol.** (a) Speed of train = $60 \times \frac{5}{18}$ mps = $16\frac{2}{3}$ mps

$\therefore$ Time taken to cross the telegraph post = $\frac{110}{16\frac{2}{3}}$ seconds

$= \frac{330}{50}$ or 6.6 seconds.

(b) Speed of train = $16\frac{2}{3}$ mps

Speed of man = $6 \times \frac{5}{18}$ mps = $\frac{5}{3}$ mps

$\therefore$ Time taken to pass the man = Length of the train/ Relative velocity

$= \frac{110}{16\frac{2}{3} - 1\frac{2}{3}} = \frac{110}{15} = 7\frac{1}{3}$ seconds.

(c) Time = Length of the train/ Relative velocity = $\frac{110}{16\frac{2}{3} + 1\frac{2}{3}} = \frac{10}{18\frac{1}{3}}$

$= 110 \times \frac{3}{55} = 6$ seconds.

(d) Time = (Length of the train + Length of platform) / Relative velocity = $\frac{110 + 240}{16\frac{2}{3}} = 350 \times \frac{3}{50} = 21$ seconds.

(e) Speed of the second train = $40 \times \frac{5}{18}$ mps = $\frac{100}{9}$ mps.

$\therefore$ Time = Sum of the length of the two trains/ Relative velocity

$= \frac{110 + 170}{16\frac{2}{3} - \frac{100}{9}} = \frac{280 \times 9}{100} = 50.4$ seconds.

(f) Speed of the second train is also 60 kmph = $\frac{50}{3}$ mps

$\therefore$ Time = Sum of the length of the two trains/Relative velocity

$= \frac{110 + 170}{\frac{50}{3} + \frac{50}{3}} = \frac{280 \times 3}{100} = 8.4$ seconds.

Example

***A thief is spotted by a policeman from a distance of 200 m. When the policeman starts a chase, the thief starts running. Speed of thief is 10 Kmph and that of policeman is 12 kmph. After how many hours the policeman will catch the thief?***

**Sol.** $t = \frac{s}{R.V.} = \frac{200}{1000 \times (12 - 10)} = \frac{200}{1000 \times 2} = \frac{1}{10}$ hr = 6 min.

Example

***A man steals a car at 1 : 30 pm & drives at 40 kmph. At 2 pm the owner starts chasing his car at 50 kmph. At what time does he catch the man?***

**Sol.** Distance covered by the thief in 1h = 40 km

Distance covered in $\frac{1}{2}$h = 20 km

Now, time taken to catch t = $\frac{\text{Distance}}{\text{Relative velocity}}$

Relative velocity = 50 – 40 = 10 kmph ($\because$ Both are moving in same direction)

$\therefore$ t = $\frac{20}{10}$ = 2 hrs.

Time = 4 p.m. ($\because$ 2 p.m. + 2)

## Boats & Streams

**Let** Speed of boat in still water = x km/hr

Speed of stream = y km/hr

Speed of boat with stream (Down Stream), u = x + y

Speed of boat against stream (Up stream), v = x – y

Speed of boat in still water, x = $\frac{1}{2}$ (u + v)

Speed of stream, y = $\frac{1}{2}$ (u – v)

**e.g.** If downstream rate = 14 km/hr

Upstream rate = 9 km/hr

Speed of man in still water = $\frac{1}{2}$(14 + 9) = 11.5 km/hr

Stream rate = $\frac{1}{2}$(14 – 9) = 2.5 km/hr.

Example

***A man can row 4.5 km/hr in still water. It takes him twice as long to row upstream as to row downstream. What is the rate of current?***

**Sol.** Speed of man in still water = 4.5 = $\frac{1}{2}$(u + v)

As u = 2v

$\therefore$4.5 = $\frac{1}{2}$ (2v + v) $\Rightarrow$ v = 3 km/hr against current

& u = 6 km/hr with current

Rate of current = $\frac{1}{2}$ (6 – 3) = 1.5 km/hr.

Circular Motion

Example

## Circular Motion with two people:

***Sachin and Saurav, as a warm-up exercise, are jogging on a circular track. Saurav is a better athlete and jogs at 18km/hr while Sachin jogs at 9 km/hr. The circumference of the track is 500 m (i.e. ½ km). They start from the same point at the same time and in the same direction. When will they be together again the first time?***

**Sol.** **Method 1:** Since Saurav is faster than Sachin, he will take a lead and as they keep running, the gap between them will also keep widening. Unlike on a straight track, they would meet again even if Saurav is faster than Sachin.

The same problem could be rephrased as "In what time would Saurav take a lead of 500 m over Sachin"?

Every second Saurav is taking a lead of $\left[18 \times \frac{5}{18} - 9 \times \frac{5}{18}\right]$m = 2.5 m over Sachin. Hence, he takes $\frac{500}{2.5}$ = 200 sec to take a lead of 500 m over Sachin.

Hence, they would meet for the first time after 200 sec.

In general, the first meeting if both are moving in the same direction and after both have started simultaneously occurs after

$$\text{Time of first meeting } t = \frac{\text{Circumference of the circle}}{\text{Relative speed}}.$$

**Do you know?**

The number of rounds the faster person makes is always one round more than the slow runner whenever and wherever they meet for the first time.

**Method 2:** For every round that Sachin makes, Saurav would have made 2 rounds because the ratio of their speeds is 1 : 2. Hence, when Sachin has made 1 full round, Saurav would have taken a lead of 1 round. Therefore, they would meet after $\frac{500}{2.5}$ sec.

[Here, $9 \times \frac{5}{18}$ m/s = 2.5 m/s is Sachin's speed.]

***Suppose in the earlier problem when would the two meet for the first time if they are moving in the opposite directions?***

**Sol.** If the two are moving in the opposite directions, then

Relative speed = 2.5 + 5 = 7.5 m/s.

[Hence, time for the first meeting = Circumference / Relative speed

$= \left(\frac{500}{7.5}\right) = \left(\frac{200}{3}\right)$ sec.

***If the speeds of Saurav and Sachin were 8 km/hr and 5 km/hr, then after what time will the two meet for the first time at the starting point if they start simultaneously?***

> **Do you know?**
>
> Problems in circular motion make use of both the relative speed and the LCM concepts.

**Sol.** Let us first calculate the time Saurav and Sachin take to make one full circle.

Time taken by Saurav = $\dfrac{500}{8 \times \dfrac{5}{18}} = \dfrac{1800}{8}$ = 225 sec.

Time take by Sachin = $\dfrac{500}{\left(5 \times \dfrac{5}{18}\right)}$ = 360 sec.

Hence, after every 225 sec, Saurav would be at the starting point and after every 360 sec Sachin would be at starting point. The time when they will be together again at the starting point simultaneously for the first time, would be the smallest multiple of both 225 and 360 which is the LCM of 225 and 360.
Hence, they would both be together at the starting point for the first time after LCM (225, 360) = 1800 sec = 0.5 hr. Thus, every half an hour, they would meet at the starting point.

**Note:** From the solution you could realise that it is immaterial whether they move in the same direction or in the opposite.

**<u>Circular motion with three people:</u>**


**Let us now discuss all the cases of motion with three people:**

***Laxman joins Saurav and Sachin, and all of them run in the same direction from the same point simultaneously. Laxman moves at 3 km/hr, Sachin at 5 km/hr and Saurav at 8 km/hr. When will all of them be together again?***

***a.*** ***for the first time?***

***b.*** ***for the first time at the starting point?***

**Sol.**

**(a)** Break the problem into two separate cases.

**In the first case,** Saurav moves at the relative speed of (8 – 5) = 3 km/hr with respect to Sachin.

At a relative speed of 3 km/hr, he would meet Sachin after every $\dfrac{500}{\left(3 \times \dfrac{5}{18}\right)}$

= 600 sec = 10 min.

**In the second case,** Saurav moves at the speed of (8 – 3) km/hr = 5 km/hr with respect to Laxman.

At a relative speed of 5 km/hr, he would meet Laxman after every $\dfrac{500}{\left(5 \times \dfrac{5}{18}\right)}$

= 360 sec = 6 min.

∴ If all the three have to meet, they would meet after every [LCM (10, 6)] min = 30 min or ½ hour. Hence, they would all meet for the first time after 30 min.

**(b)** If we need to find the time after which all of them would be at the starting point simultaneously for the first time, we shall use the same method as in the case involving two people.

At a speed of 8 km/hr, Saurav takes 225 sec. to complete one circle.
At a speed of 5 km/hr, Sachin takes 360 sec. to complete one circle.
At a speed of 3 km/hr, Laxman would take 600 sec. to complete one circle.
Hence, they would meet for the first time at the starting point after
LCM (225, 360, 600) sec. = 1800 sec.